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WALL-THICKNESS EFFECT ON THE THERMOHYDRAULIC STABILITY OF A HOMOGENEOUS TWO-PHASE FLOW

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An approximate analytical solution of the problem of thermohydraulic stability of a homogeneous two-phase flow is obtained with allowance for the effect of the channel-wall thickness.

As is known [1, 2], when a homogeneous two-phase flow moves in a heated channel, there can develop thermohydraulic instability, which is related to the "lagging interaction" of the flow rate, density, and pressure disturbances and has the following mechanism. A random disturbance of the flow rate at the beginning of the channel changes the amount of heat received by unit mass of the heat carrier. This results in a change in the vapor content and, consequently, the density of the two-phase flow. Since the flow-rate disturbances propagate practically instantaneously and the density disturbances propagate approximately at the velocity of the heat carrier, "kagging" pressure disturbances are generated at the channel outlet. The pressure drop between the inlet and the outlet of the channel is set externally and is independent of the hydrodynamic conditions of the heat carrier; therefore a feedback develops in the flow that causes a pressure change at the channel inlet that initiates a new disturbance of the flow rate and so on.

A one-dimensional representation of the problem of thermohydraulic instability includes the equations [3] of: continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0 ,$$

motion

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial z} + \frac{\xi}{2D} \rho u^2 = - \frac{\partial p}{\partial z}$$

and energy

$$\rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial z} = \frac{4q}{D} .$$

Use of the linear dependence of the specific volume of the homogeneous two-phase flow on its enthalpy allows, after some transformations, the initial system of the equations to be written in the form [4]

$$\frac{\partial u}{\partial z} = \Omega , \tag{1}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial z} = v , \tag{2}$$

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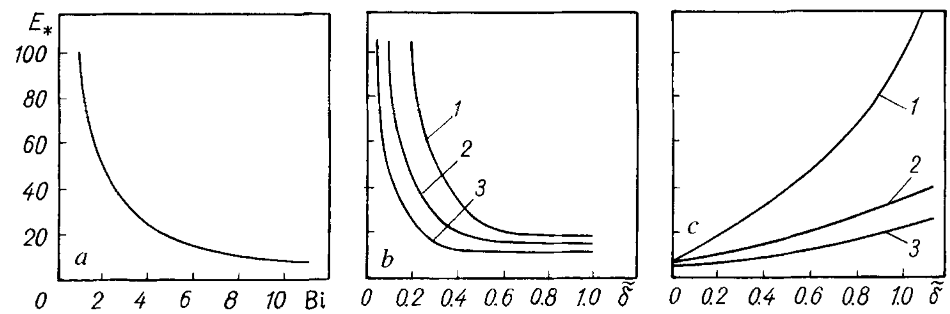

Fig. 1. Expansion parameter of the flow at the stability boundary vs. Biot number for a thick wall (a) and vs. dimensionless wall thickness for the cases T_∞ = const [b: 1) Bi = 5; 2) 10; 3) 20] and q_∞ = const [c: 1) Bi = 1; 2) 2; 3) 5].

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + \frac{\xi}{2\Omega}u^2 = -v\frac{\partial p}{\partial z}. \tag{3}$$

Here $\Omega = 4q/r\rho_v$; r is the specific heat of the phase transition; ρ_v is the density of the saturated vapor.

In the majority of works [1-4], the methods of the linear theory of automatic control [5] are used for analyzing the thermohydraulic instability. The present investigation is aimed at determination of the stability boundary of a homogeneous two-phase flow with allowance for the effect of the channel-wall thickness. For this, use is made of results of the solution [6] of the boundary-value problem for a two-dimensional unsteady equation of heat conduction in the wall with a periodic boundary condition of the third kind.

The known procedure of linear analysis of hydrodynamic instability [7] leads to the following system of equations for pulsating quantities:

$$\frac{\partial u'}{\partial z} = \frac{\Omega q'}{\bar{q}}, \tag{4}$$

$$\frac{\partial v'}{\partial t} + \bar{u}\frac{\partial v'}{\partial z} + u'\frac{\partial \bar{v}}{\partial z} = \Omega v', \tag{5}$$

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial z} + u'\frac{\partial \bar{u}}{\partial z} + \frac{\xi}{D}\bar{u}u' + \bar{v}\frac{\partial p'}{\partial z} + v'\frac{\partial \bar{p}}{\partial z} = 0. \tag{6}$$

As boundary conditions for (4)-(6), the conditions of undisturbance for the specific volume at the channel inlet

$$v_1' = 0 \tag{7}$$

and pressure drop along the channel

$$p_2' - p_1' = 0 \tag{8}$$

are prescribed.

A solution of the system of equations (4)-(8) leads to the following dispersion relation of the linear analysis of stability:

$$(2-\omega)(1+A)[A-2(1+A)\omega]\left(E^{\frac{2+A}{1+A}}-1\right)+A(2+A)(E^{2-\omega}-1)=0, \tag{9}$$

where $E = \bar{u}_2/\bar{u}_1 = \bar{v}_2/\bar{v}_1 = 1 + \Omega l/\bar{u}_1$ is the "expansion parameter" of the homogeneous two-phase flow between the inlet and the outlet; A is the "contingency parameter" dependent on the thermophysical properties and the wall thickness of the channel as well as on the conditions of external heat supply [6]: A = Bi tanh $\tilde{\delta}$ at T_∞ = const; A = Bi cot $\tilde{\delta}$ at q_∞ = const; $\mathrm{Bi} = \bar{\alpha}/\sqrt{\Omega \lambda_w \rho_w c_w}$ is the Biot number; $\tilde{\delta} = \delta\sqrt{a_w/\Omega}$ is the dimensionless wall thickness.

The sought dispersion relation is the dimensionless frequency of pulsations

$$\omega = \gamma + i\beta ,$$

where the characteristic frequency $\Omega = 4q/r\rho_v D$ is adopted as a scale.

Equality (9) is equivalent to two relations determining the real γ and imaginary β parts of the complex quantity ω. Here, β characterizes the frequency of periodic pulsations of the disturbed parameters, γ describes the flow mode: stable ($\gamma < 0$), neutral ($\gamma = 0$), or unstable ($\gamma > 0$). Assuming $\gamma = 0$ [$\omega = i\beta$ in (9)], we arrive at the system of equations determining the stability boundary:

$$2\,[A - (1 + A)\,\beta_*^2] + (1 + A)\,E_*^{\frac{A}{1+A}} \cos(\beta_* \ln E_*) = 0 , \tag{10}$$

$$5\beta_* + E_*^{\frac{A}{1+A}} \sin(\beta_* \ln E_*) = 0 , \tag{11}$$

the approximate solution of which is of the form

$$2\beta_*^2 = E_*^{\frac{A}{1+A}} , \tag{12}$$

$$E_*^{\frac{A}{1+A}} \ln^2 E_* = 8\pi^2 . \tag{13}$$

Consider the effect of the channel-wall thickness on the position of the stability boundary in accordance with relation (13).

For the case of a thick wall ($\tilde{\delta} \geq 1$), there is a region of self-similarity with respect to the thickness. Here, the influence of external heat supply is absent, while the parameter of flow expansion at the stability boundary depends on the Biot number (see Fig. 1a).

For the case T_∞ = const, with decrease in the wall thickness the stability boundary in the limit shifts to infinity. With increase in the wall thickness, the quantity E_* asymptotically tends to a maximum dependent on the Biot number (see Fig. 1b).

For the case q_∞ = const, as the wall thickness decreases to zero the expansion parameter of the flow at the stability boundary attains its minimum possible value $E_* \approx 9.3$. With increase in the wall thickness, the quantity E_* reaches an asymptotics similar to the case T_∞ = const (see Fig. 1c).

For all the cases considered, the frequency of periodic pulsations of the disturbed parameters at the stability boundary β_* is determined from relation (12).

Thus, the minimum of the thermohydraulic stability is attained in the limit of zero thermal conductivity of a thick wall and in the limit of zero thickness of the wall at q_∞ = const. With the thermal conductivity of the thick wall tending to infinity and with the wall thickness tending to zero at T_∞ = const the limit of thermohydraulic stability tends to infinity ($E_* \to \infty$).

Note that the development of disturbances (with allowance for the interaction of their amplitudes and phases) after stability loss by the flow can be analyzed only within the framework of the statement and solution of the corresponding nonlinear problem [8].

NOTATION

t, time; z, longitudinal coordinate; u, longitudinal velocity; ρ, density; v, specific volume; λ, thermal conductivity; a, thermal diffusivity; c, specific heat; ξ, coefficient of hydraulic resistance; p, pressure; h, enthalpy; D, hydraulic diameter of the channel; l, channel length; δ, thickness of the channel wall; q, heat-flux density; α, heat-transfer coefficient; Ω, scale of the pulsation frequency; ω, dimensionless pulsation frequency. Superscripts: line, stationary value; prime, pulsating value. Subscripts: 1, channel inlet; 2, channel outlet; w, channel wall, ∞ , external (heated) channel surface; *, stability boundary.

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